

THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Aerospace Engineering and Engineering Mechanics

E M 386K ANALYTICAL METHODS I (15310)

ASE 380P 1-ANALYTICAL METHODS I (14865)

COE 371 APPLIED MATHEMATICS I (15085)

FALL 2023

SYLLABUS

Unique Number: 15310, 14865, 15085

Instructor: **Tan Bui**
Office: ASE 4.220, telephone numbers: 512-471-8176, email address:
tanbui@oden.utexas.edu. Office hours from 2-3pm every Thursday.

Time: **TTH 9:30—11:00**

Location: CBA 4.326

Optional Recitation: TBA

Teaching Assistant: Krishnanunni Giri (3SEo2B, 3rd Floor, POB Building) Office Hours: Wed 1-2pm In-person. Email address: krishnanunni@utexas.edu

Web Page: <https://piazza.com/utexas/fall2023/ase380p1> all registered students are required to register for this class. Every student is required to register for this Piazza page. Failing to register may miss important information/communications about the lecture notes, homework, Exam, Q/A, etc. Students are strongly encouraged to answer questions from other students on Piazza.

Catalog Description:

15310: Introduction to modern mathematics, real analysis of functions of one variable, linear algebra, elements of real analysis of functions of many variables, calculus of variations. Aerospace Engineering 380P (Topic 1) and Engineering Mechanics 386K may not both be counted.

14865: Basic topics in real and complex analysis, ordinary and partial differential equations, and other areas of applied mathematics with application to applied mechanics.

15085: Restricted to computational engineering majors. Subjects include real analysis of functions of one variable, linear operator theory, and ordinary differential equations.

Course Objectives:

The class will cover advanced topics in logic, series, linear algebra and linear operators, Ordinary differential equations (ODEs), and numerical methods.

Prerequisites:

Graduate standing and consent of instructor. Upper-division standing; Mathematics 427J, 427L, and 362K with a grade of at least C- in each

Knowledge, Skills, and Abilities Students Should Have Before Entering This Course:

Calculus, Linear algebra, and basic numerical methods covered in Engineering Computation 311K

Knowledge, Skills, and Abilities Students Gain from this Course (Learning Outcomes):

Students will learn a deeper understanding about series, ODEs, numerical methods, vector spaces, and new knowledge about these topics

Impact on Subsequent Courses in Curriculum:

Students have to take this before taking Applied Mathematics II and Analytical Methods II

Relationship of Course to Program Outcomes and ABET Program Criteria Achieved:

(<https://utexas.box.com/v/asecoe-abet-criteria>)

Topics:

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| 1 | Review of the Basics | 7 |
| 1.1 | Elementary Logic | 7 |
| 1.2 | Notation for Sets Relations | 9 |
| 1.3 | Sets, Functions, and Sequences in \mathbb{R} | 10 |
| 1.4 | Limits and Continuity | 11 |
| 1.5 | Sequences of numbers (recap) | 15 |
| 1.6 | Sequence of functions | 15 |
| 1.7 | Infinite series | 23 |
| 1.8 | A quick check on the divergence | 26 |
| 1.9 | Function series | 26 |
| 2 | Fourier Series | 31 |
| 2.1 | Fourier Series (Review) | 31 |
| 2.1.1 | A few properties for $L^2(\Omega)$ | 32 |
| 2.2 | Uniqueness of Fourier series | 37 |
| 2.3 | Term-wise Integration of Fourier series | 37 |
| 2.4 | Term-wise Differentiation of Fourier series | 39 |
| 2.5 | General periodicity | 40 |
| 3 | Vector Spaces | 41 |
| 3.1 | Metric Space | 46 |
| 3.2 | Convergence in vector space | 48 |

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| 4 | Linear Operator and Map (Generalization of Matrices) | 51 |
| 4.1 | Linear operator | 51 |
| 4.2 | Inner product | 56 |
| 4.3 | Adjoint operator | 58 |
| 5 | Solvability of linear system of equations | 61 |
| 5.1 | Solvability of linear equation system | 61 |
| 5.2 | Necessary condition for solvability | 62 |
| 5.3 | Uniqueness of solution | 66 |
| 5.4 | Summary of existence and uniqueness | 66 |
| 5.4.1 | Some additional problems on existence and uniqueness | 67 |
| 5.5 | Rank-Nullity Theorem | 70 |
| 6 | Eigen Value Problem | 75 |
| 6.1 | Eigen value problem | 75 |
| 6.2 | Eigen value problem in finite dimensional case | 78 |
| 6.3 | Sturm-Liouville as a self-adjoint operator in $L^2(a, b)$ | 79 |
| 6.4 | Singular Value Decomposition (SVD) | 84 |
| 6.4.1 | Application to Matrices | 86 |
| 6.4.2 | Rank-Nullity Theorem Revisited | 87 |
| 6.4.3 | Fundamental Theorem of Linear Algebra | 89 |
| 7 | Theory of Ordinary Differential Equations | 91 |
| 7.1 | Challenges of Ordinary differential equations (ODE) | 91 |
| 7.1.1 | Non existence of solution | 91 |
| 7.1.2 | Non-uniqueness of solution | 91 |
| 7.1.3 | A solution may not be defined for all x | 92 |
| 7.2 | Conventions/terminology | 92 |
| 7.3 | The Cauchy Problem: Existence and Uniqueness | 92 |
| 7.4 | Homogeneity of order 0 | 95 |
| 7.5 | Exact equations | 97 |
| 7.5.1 | Integrating factor to achieve exactness | 99 |
| 7.6 | Second order linear ODEs | 100 |
| 7.7 | Linear independence/dependence and the Wronskian | 101 |
| 7.8 | Linear nonhomogeneous second order ODE | 104 |

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| 7.9 | Oscillatory behaviour of solution | 106 |
| 7.10 | Analysis of 2×2 systems and nonlinear second order equation | 108 |
| 7.10.1 | Planar Hamiltonian systems | 108 |
| 7.10.2 | Lotka–Volterra prey-predator systems | 110 |
| 7.10.3 | The case of fishing | 111 |
| 7.10.4 | Susceptible-Infectious (SI) model for infectious diseases | 112 |

Professionalism Topics:

N/A

Design Assignments:

N/A

Laboratory Assignments:

N/A

Computer:

The last part of the class will involve programming in Matlab

Text:

Lecture notes will be provided. This book is useful.

Advanced Engineering Mathematics 2nd Edition

By Greenberg

Class Format:

In-Person Classes (Face-to-face) on Tues and Thurs from 9:30-11am and **one recitation hour per week (to be determined)**

Class Schedule:

Please see **Topics**.

The class will have

- A) Weekly-biweekly homeworks
- B) Three midterm take-home exams (09/22, 10/27, and 12/1)
- C) No final exams

Class Outline:

Please see **Topics**.

Grading:

Homework: 55% (Each assignment might not be weighted the same)

Exams: 15% each (45% total)

***Note that attendance is not considered in a student's final grade; however, regular attendance is expected.**

A : Grade ≥ 90

A- : $85 \leq \text{Grade} < 90$

B+: 80 <= Grade < 85
B : 75 <= Grade < 80
B- : 70 <= Grade < 75
C+: 65 <= Grade < 70
C : 60 <= Grade < 65
C- : 55 <= Grade < 60
D : 50 <= Grade < 55
F : Grade < 50

Homework Policy:

There will be approximately 7-10 homework assignments in the semester. Students will be given a minimum of one or two weeks to complete each assignment. Assignments **must be submitted electronically via canvas. Students are responsible for submitting and checking the correct files (no excuse for submitting wrong files).**

Assignments can be turned in up to 3 days late with a loss of an additional 10% (of graded points) for each day the assignment is late. After 3 days late, 0 will be assigned to the assignment. No exceptions to the homework policy will be made without prior instructor approval, that is, students must contact Dr. Bui to provide PROOF for exception prior to the due day.

Students are expected to **complete their assignments independently**. Students are encouraged to discuss general class topics amongst themselves; however, any sharing of code or copying or plagiarism **will be considered an act of academic dishonesty and will be dealt with severely under the Code of Conduct of the university**. In addition, it is expected that students make an effort to produce organized and professional looking assignments.

Examinations:

The class will consist of **three open-book 2-hour midterm take-home exams** according to the scheduled given above. Each exam will cover the topics preceding the exam. **If you have a conflict with any of the class exams, you must notify the instructor immediately at least one week ahead of any exam.**

Attendance:

Attendance is required, though not checked. **Students who wish set-up alternate arrangements to discuss a missed lecture must have prior approval from the instructor or a note verifying reason for absence.**

Office Hours:

Instructor: Office: ASE 4.220, telephone numbers: 512-471-8176, email address: tanbui@oden.utexas.edu. Office hours from 2-3pm every Thursday.

TA: Office 3SEo2B, 3rd Floor, POB Building. Office Hours: Wed 1-2pm In-person. Email address: krishnanunni@utexas.edu

Important Dates:

Three midterm take-home exams (09/22, 10/27, and 12/1)

Special Notes:

The University of Texas at Austin provides upon request appropriate academic adjustments for qualified students with disabilities. For more information, contact the Office of the Dean of Students at 471-6259, 471-4641 TDD or the Cockrell School of Engineering Director of Students with Disabilities at 471-4321.

It is your responsibility to read all Canvas announcements from Dr. Bui and the TA, where important messages are sent to you (every registered student should get messages automatically). We are extremely busy, thus if we spend time writing something for you it must be important. And you will take all the responsibilities if you miss important things because you don't read the announcements/syllabus. Questions whose answers are provided in Syllabus, Canvas Announcements, or Piazza Q/A will be referred to these resources.

Evaluation:

Note that the Measurement and Evaluation Center forms for the Cockrell School of Engineering will be used during the last week of class to evaluate the course and the instructor. They will be conducted in an electronic format for Summer 2022. You may also want to note any other methods of evaluation you plan to employ.

Sharing of Course Materials is Prohibited

No materials used in this class, including, but not limited to, lecture hand-outs, videos, assessments (quizzes, exams, papers, projects, homework assignments), in-class materials, review sheets, and additional problem sets, may be shared online or with anyone outside of the class unless you have my explicit, written permission. Unauthorized sharing of materials promotes cheating. It is a violation of the University's Student Honor Code and an act of academic dishonesty. I am well aware of the sites used for sharing materials, and any materials found online that are associated with you, or any suspected unauthorized sharing of materials, will be reported to Student Conduct and Academic Integrity in the Office of the Dean of Students. These reports can result in sanctions, including failure in the course.

Class Recordings

Class is in-person and under some rare occasions (such as when the instructor is out of town), the lectures are recorded. Class recordings are reserved only for students in this class for educational purposes and are protected under FERPA. The recordings should not be shared outside the class in any form. Violation of this restriction by a student could lead to Student Misconduct proceedings. Guidance on public access to class recordings can be found [here](#).

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| August 21 | First day of classes |
| August 21 - 24 | Fall registration for all students |
| August 24 | 4th class day |
| | Last day to add a class without permission |
| | Tuition payment deadline is 5:00 p.m. CST for graduate and I |
| August 25 - September 6 | Fall registration for all students (will require either department |
| September 4 | Labor Day holiday; no classes held |

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| September 6 | 12th class day |
| | Last day to add a class (approval required) |
| | Last day to drop a class without permission |
| | Tuition payment deadline is 5:00 p.m. CST for anyone who re |
| | Payment for added classes ("Add Bill") is due 11:59 p.m. Any placed on an installment plan. |
| October 13 | Payment for remaining tuition balances due by 11:59 p.m. CS |
| October 23 | Last day an undergraduate may: Q-drop a class; withdraw; ch |
| | Last day to apply for an undergraduate degree |
| October 30 - November 10 | Spring 2024 registration |
| November 14 | Spring 2024 tuition bills distributed to students |
| November 17 | Last day a doctoral candidate may hold a dissertation defense |
| | Last day to apply for a graduate degree |
| November 20 | Last day a graduate student may change registration in a clas |
| November 20 - 25 | Fall break / Thanksgiving; no classes held |
| December 1 | Last day to submit master's report, recital, thesis, doctoral dis |
| December 4 | Last class day |

Prepared by: Tan Bui

Date: 08/15/23