ME 449 – Redesign and Prototype Fabrication

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Professor: Frank Pfefferkorn

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Team J:

Ahmed Khalil, ahmed.khalil@wisc.edu

Josh Richlen, jrichlen@wisc.edu

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Introduction

Stirling engines are simple, easily constructed engines that are able to run on virtually any source of heat, even at relatively low levels. They are best known for their potential to be more efficient than a gas or diesel engine or even as an alternative source of clean fuel. The original Stirling engine was invented and patented by Robert Stirling on September 27, 1816 and first used in 1818 as a pumping device to push water into a quarry. During the late 1980's and the early 1990's Professor Senft of the University of Wisconsin took up the idea of low temperature differential Stirling engines. The first models he produced were Ringbom engines, where there is no direct connection between the flywheel and the displacer, the Ringbom engine is reliant on the changing pressure inside the main chamber to move the displacer back and forth. [1]

The Ringbom engine is a unique Gamma type engine in which the piston is connected to a crankshaft, however, the displacer is driven only by the gas pressure forces on a displacer piston rod [1]. This is the engine type that this project is focused on and can be shown in Figure 1.

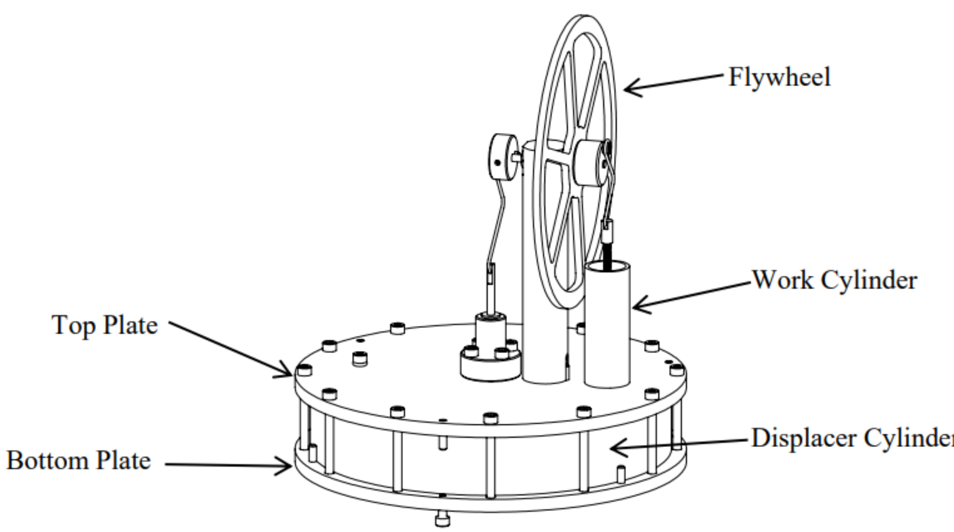

Figure 1: Schematic of gamma-type Stirling engine used in the course [2]

This project consisted of a baseline section and a redesign section. The baseline section involved manufacturing and assembling a Stirling Engine Gamma configuration from a set of instructions. One of the purposes of the baseline engine was to get the students acquainted with various manufacturing methods, such as: lathes, mills, 2-axis CNC controllers, and Waterjet cutting. Another purpose was to get the students acquainted with how the engine operates to help them identify areas of improvement when redesigning the engine. The redesign section involved identifying key stakeholders, understanding their needs, and determining important design requirements. After a set of goals had been set, the following steps involved brainstorming potential design solutions, utilizing engineering background knowledge to quantify improvements, and iterating designs until a substantial improvement had been achieved. Once the designs are finalized, the next step was to develop a schedule to ensure that all requirements are met in order of importance and that the tasks were feasible given the short time frame. Finally, each part in the redesign was fabricated or purchased, the engine was assembled, and each component was tested individually to determine its effect on the engine's overall performance.

This remainder of the report will discuss Team J's main design changes and the reasoning behind each change.

Design Requirements

When determining the design requirements for this project, the team first identified and ranked all of the stakeholders in order of highest importance while including a short description for reasoning. Table 1 summarized all of the important customers involved in this project.

Table 1: List of customers for Stirling Engine Project with description to reason their importance

Rank / Customer #	Customer Name	Customer Description
1 (most important)	Design Team/Student	By designing the product, students will gain valuable hands-on manufacturing and product design experience. The more robust and sophisticated the designs are, the more the students will learn and be able to apply that knowledge to their future careers. By carrying out the competition at the end of the semester, students are incentivized to test the limits and create better designs which will just build experience.
2	Instructors	By offering this class to mainly upperclassmen, instructors will not only be able to get valuable feedback regarding how this specific course could be improved, but also how other courses in the mechanical engineering curriculum could be improved. If the students enjoy the course, they might try to build a collaboration between their future employers and that course (e.g. Caterpillar hosts an engine redesign for one of their Stirling engines).
3	Other Students in ME 449	These are the people who will be assessing our Stirling engine's design aesthetics and looks. We have to ensure that we target their specific tastes and get insight into what components on the Stirling engine we should focus on redesigning.
4	Future students' experiences	If more instructors take note of the way this class is organized, there might be a shift in the mechanical engineering curriculum towards more design-oriented courses. This could potentially improve many future students' experiences and equip them with a wider range of skills upon graduation. Students will spend less time sitting in a lecture hall and spend more time developing the skill sets required to become successful engineers.
5	University's research departments	By going through the iterative process of designing Stirling engines, students could potentially realize a design that

		could be used by a research group at UW Madison and lead to a breakthrough. While the general engine design probably won't be too valuable, a specific design specification might create inspiration for a better design in a research lab.

After identifying all of the customers, a list of requirements was created for each customer. The list details what each customer is looking for or interested in and the design requirements needed to satisfy their needs, as shown in Table 2.

Table 2: Design requirements for each customer and how important the requirements are.

Requirement #	**Description**	**Customers**	**Importance (1 – 5)**
1	<u>A fast Stirling Engine design</u>	● Design Team/Student ● Instructors ● University research departments	5 *Speed (somewhat related to horsepower) is related to how efficient and effective the engine is*
2	<u>A robust Stirling Engine design that produces consistent results</u>	● Design Team/Student ● Instructors ● University research departments	4 *This will make the testing process easier as the Engine's results will be more precise and consistent, making it easier to find trends which could be optimized in future design*
3	<u>Build quality of Stirling Engine Design</u>	● Design Team/Student ● Other Students in ME 449	4 *A better build quality will lead to an overall more efficient engine*
4	<u>Aesthetics of Stirling Engine Design</u>	● Design Team/Student ● Other Students in ME 449	3 *Engines are more about performance than looks, however, our grade is affected by aesthetics*

Objectives / Goals

There are two primary objectives for this redesign project, the first being to achieve the highest possible points in the performance competition by maximizing speed and meeting our audience's needs for build quality/aesthetics. The second goal is to develop as many skills as possible that could be transferred into our future careers. By utilizing various manufacturing methods, working with different engineering software, and embracing the iterative process of design, both team members would gain invaluable experience of implementing their strong engineering knowledge. "Tell me and I forget. Teach me and I remember. Involve me and I learn".

In order to achieve the maximize speed, the team decided to focus on following re-design criteria:

- Reducing frictional contact at the links
- Limiting fluid losses from the system
- Minimizing conduction between the top and bottom plates
- Smoothening motion of the engine

While the speed requirement could be determined through a series of engineering analyses, the aesthetics and build quality required the team to carry out surveys to identify customer requirements. The re-design criteria that the team focused on for aesthetics/build quality was:

- Identifying customer needs and incorporating it into the design

Prototype Changes/Improvements

Given the re-design criteria determined in the objectives/goals section of the report, a variety of design changes were made and incorporated into the re-designed engine. The main components that were re-designed include: the cylinder base, the flywheel, the top plate, bearings, the counter balances on the displacer hub, and the bolts holding the two plates together.

Cylinder Base

In the baseline design, the cylinder base was internal to the power piston cylinder. Using an O-ring, the power piston cylinder was pressed down over the cylinder base to create a seal. While this design functioned to prevent air from escaping at this location, the passageway for air to move through was smaller than the diameter of the power piston itself. This meant that when air was pushed down into the lower portion of the Stirling Engine, the air went through a contraction. Conversely, when air was pushed from the lower portion into the power piston cylinder, the air went through an expansion. Both the down and up strokes of the piston resulted in fluid flow loss due to the pressure drop.

Instead of having the cylinder base seal the power piston cylinder from the inside, the redesigned cylinder base eliminated the contraction/expansion by sealing the cylinder from the outside. By incorporating an O-ring on the outside of the cylinder, the air-tight seal was maintained. Additionally, the part still functioned as a passageway for air to contact the power piston but without the additional flow restriction. The reduced pressure difference across the cylinder base was used more efficiently to move the power piston. Unfortunately, eliminating the constriction of the cylinder base led to an insignificant increase in rotational speed.

Figure 2 below illustrates the original cylinder base and modified cylinder base design.

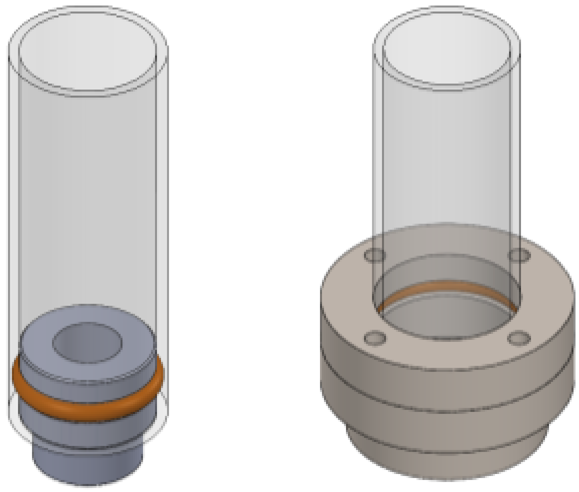

Figure 2: A: Original cylinder base where glass cylinder is mounted internally. B: Redesigned cylinder base where glass cylinder is mounted externally.

As a result of this design change, there is a difference in how air moves through the cylinder. Originally, air must go through a constriction, where now it can freely flow. In each cylinder, there is a head loss to the flow of air which results in a pressure drop. Whichever cylinder base design has a lower pressure drop due to a head loss, will be the most efficient to the stirling engine.

Calculations outlining the pressure drop are outlined in Appendix A. Equations for these calculations were taken from Fluid Dynamics by Frank White [3]. The resulting pressure drop is shown in Table 3.

Table 3: Theoretical pressure drop across original and redesigned cylinder base.

	Original	Redesign
Downward Stroke	ΔP_contraction = 0.02489 Pa	ΔP_downward = 0.005177 Pa
Upward Stroke	ΔP_expansion = 0.02505 Pa	ΔP_upward = 0.005177 Pa

As shown in Table 1, the redesigned cylinder base reduced the pressure drop across the part by more than 4 times. The redesigned part was placed on the final stirling engine. Part drawings for the manufactured parts necessary can be found in appendix B.

Flywheel

One of the major issues with our baseline design was that the flywheel would have significant wobble while spinning. This was a result of the aluminum flywheel being slightly deformed during Waterjet cutting and the hub of the flywheel not fitting correctly into the flywheel's hole. In addition to the flywheel wobbling, the engine was highly reliant on the kickstart required to overcome its static friction. To ensure that the engine ran smoothly and reliability during the day of testing, the team decided to redesign the flywheel.

To ensure that the flywheel hub would be tightly fit inside of the flywheel, the material of the flywheel was changed such that the flywheel would be able to deform to meet the size of the hub. The optimal material for this need given its fast manufacturability and low cost was 3D printing Polylactic Acid, also known as PLA. While the 3D print had a higher chance of cracking when deforming, it only cost $2 and took 2 hours to rebuild, making it great for fast prototyping and testing. As shown in the flywheel's part drawing, the 3D printed flywheel section had a center hole of 0.610" which only required a 0.623" reamer to fit in the flywheel hub.

After pressing the hub into the 3D part, the two pieces were very well aligned with significantly less wobble. Unfortunately, the whole part could not be solely made out of PLA due to its very low density, 1.25 g/cm^3 [4], which would drastically affect the flywheel's moment of inertia. The moment of inertia of the flywheel is a measure of resistance to torque applied on a spinning object so to ensure that engine does not start decelerating, a high moment of inertia was required. As shown in Equation 1 [5], the moment of inertia for a disc is a product of the mass and its distance from the center of the disc, meaning that to maximize inertia, one needs to have lots of mass far away from the center of the disc.

$$I = 0.5 * m * R^2$$

Equation 1

To increase the 3D flywheel's moment of inertia, it was decided to add an aluminum disc surrounding the 3D printed flywheel since it could easily and quickly be fabricated using the Team Lab's Waterjet cutter. Figure 3 shows the baseline flywheel Solidworks design and the re-designed flywheel Solidworks design.

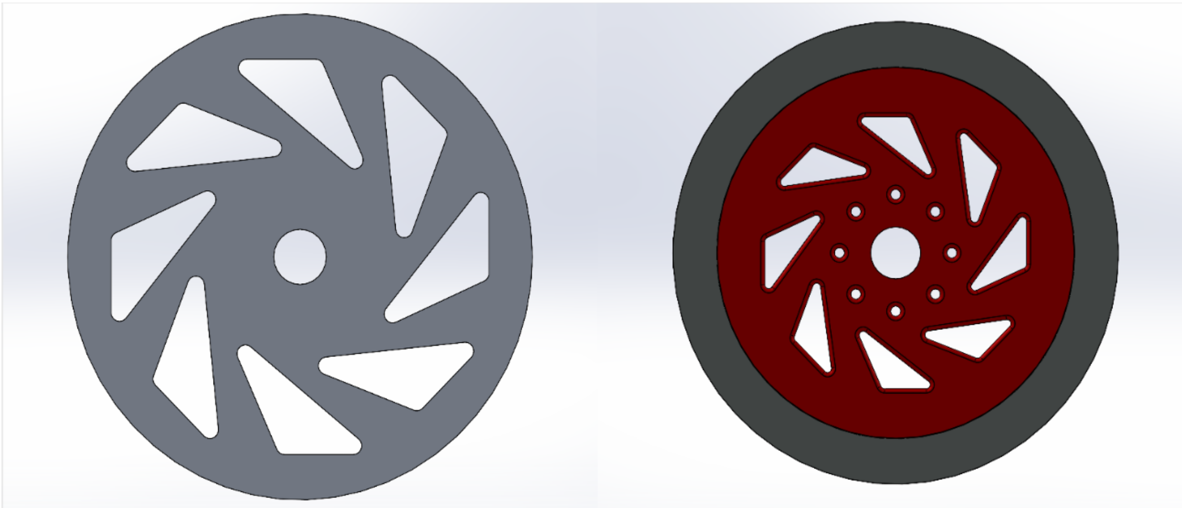

Figure 3: Solidworks designs for baseline (left) and re-designed (right) flywheels.

The baseline design had a mass of 0.31 [g] and a moment of inertia of 1.26 [in-lb] while the redesign had a mass of 0.27 [g] and a moment of inertia of 1.31[in-lb]. By changing the material and redistributing it around the flywheel, the mass was reduced by 15% while increasing the moment of inertia slightly. The lower mass is beneficial on testing day because it makes the engine easier to spin, making it less reliant on the initial kickstart. The lower mass also increases the flywheel's acceleration, allowing it to reach its maximum speed earlier. By decreasing the time to reach maximum speed, the engine will have a higher speed after 1 minute of spinning during testing day.

In addition to satisfying the engineering requirements, the flywheel redesign also satisfies the build quality and aesthetics requirements. The requirements were met by having a red and black flywheel with the aluminum disc having a horizontal brushed surface finish.

Top Plate and Aesthetics

An improvement to the aesthetics of our stirling engine was a focus for the redesign. During the initial build, our engine fell below the class average for aesthetic scores. To improve this, we sent out a survey to those attending our graduation party, where the stirling engine will be on display. We asked 3 questions on this survey: “What color do you want on the top plate?”, “What are your favorite Wisconsin icons that you would like to see on the engine?”, and “What is your favorite metal finish?”. The results from the survey are shown in the figures below.

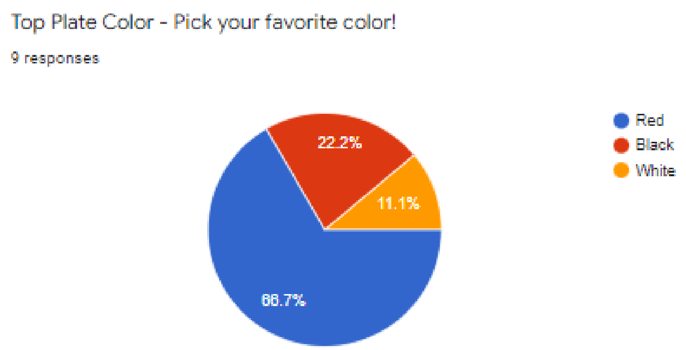

Figure 4: Survey results for top plate color.

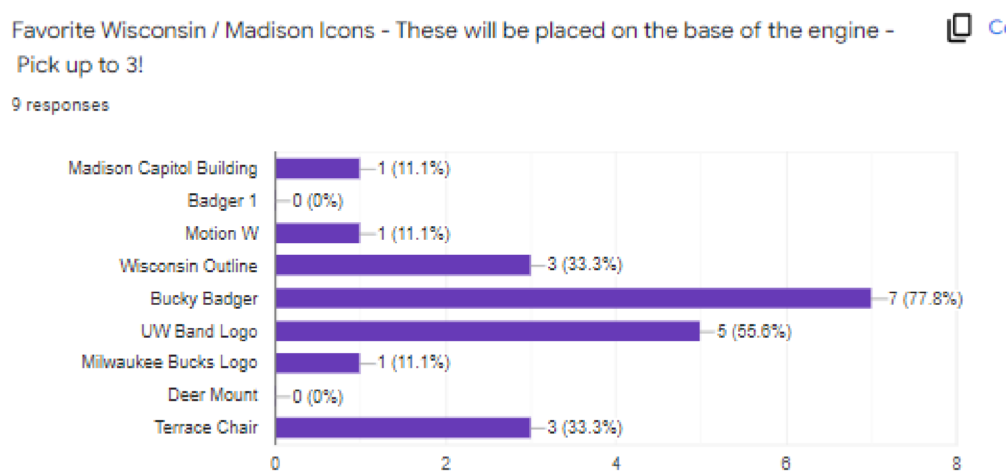

Figure 5: Survey results for favorite Wisconsin icons.

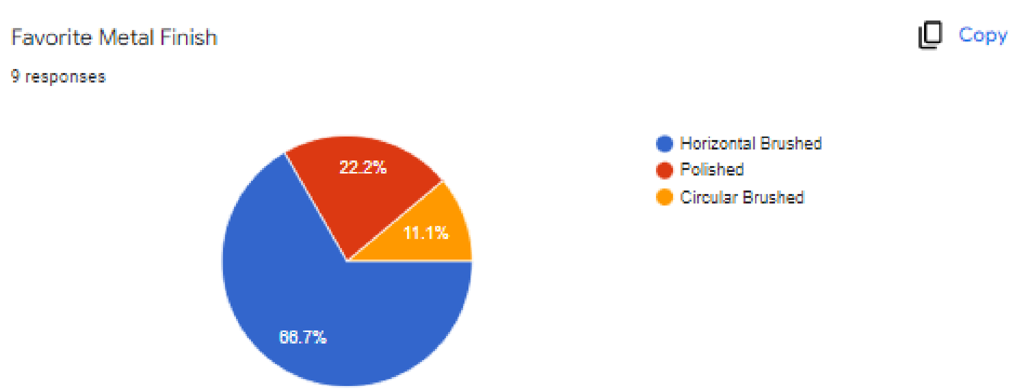

Figure 6: Survey results for favorite metal finish.

The results of the survey indicated that the attendees of our graduation party wanted to see a red top plate with a badger, a cutout of wisconsin, and a UW Marching Band logo, with a horizontal brushed metal finish on the fly wheel. Listening to the responses, the final aesthetics with these changes are shown in the figures below.

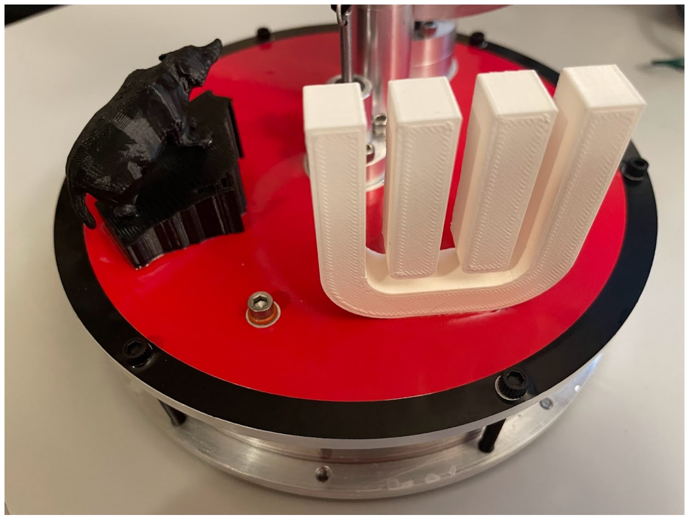

Figure 7: Top plate with red color and badger, Wisconsin, and UW Marching Band logo 3D prints.

Figure 8: Horizontal brushed metal finish on flywheel.

After following these recommendations from the survey, our aesthetics surpassed the class average of 9.55 and scored a 9.89, one of the highest in the class.

Bearings and Links

During the initial build of the stirling engine, it was clear that having properly manufactured links with low friction was essential to making the engine run. The slightest bend in the link could be the difference between a fast stirling engine, and one that does not run at all. For this reason, it was important that we could rely on a low friction link as it spun around the connecting point. In the original design, a steel wire was bent into a circle so that it could rotate around a steel screw. There were two issues with this design of linkage. First, the steel on steel coefficient of friction is high, at around .42 [6]. Second, the thread on the screw made the link bind as the flywheel rotated, further reducing speed. To eliminate both of these problems, a steel shoulder bolt with Rulon J sleeve bearing was used with the addition of a 3D printed link. The coefficient of friction between the sleeve bearing and shoulder bolt was reduced to a typical value between 0.12-0.2, immediately reducing the friction of this interface in half [7]. The shoulder bolt also eliminated the binding characteristics of the original design. The shoulder bolt, Rulon J sleeve bearing, and 3D printed link are shown below in Figure 9.

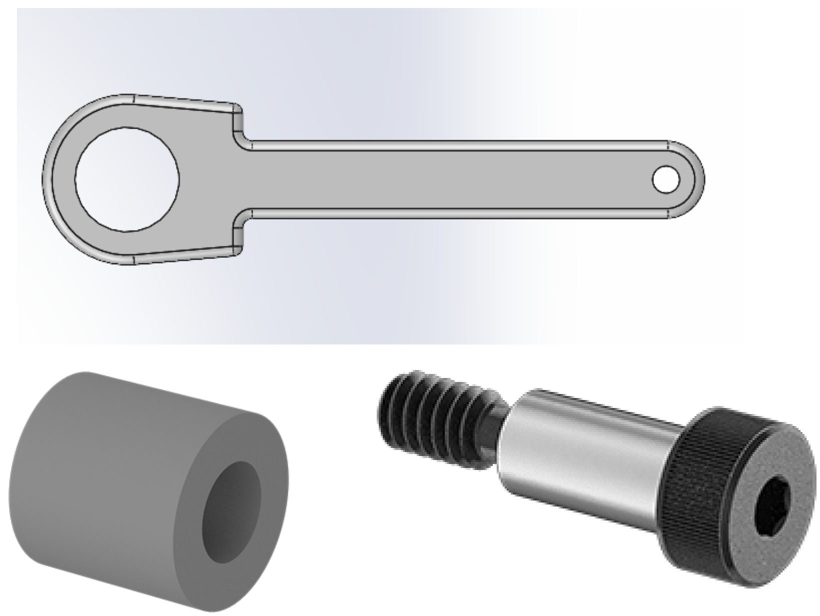

Figure 9: 3D printed link (top), Rulon J sleeve bearing (bottom left), shoulder bolt (bottom right).

This redesign was the most important improvement to the speed of our engine, increasing it from 87 to 155 rpm.

Counter Balances on Displacer Hub

On each side of the flywheel post, there are bushings in which the power piston or displacer are attached to. A moment is applied onto the displacer due to the addition of a mass that is offset by a distance. As the engine spins, it is off balanced from the force of gravity pulling down on the mass of the displacer or power piston. Since the mass of the displacer, 27.2g, is much larger than that of the power piston, 5.54g, the addition of a counter balance will be done on the displacer side.

Without a counter balance, the system experiences accelerations and decelerations throughout its rotational motion, meaning that it speeds up for half of its rotation, and slows down for the other half. Creating a balanced rotation will increase the speed of the engine. These accelerations and decelerations are shown in Figure 10.

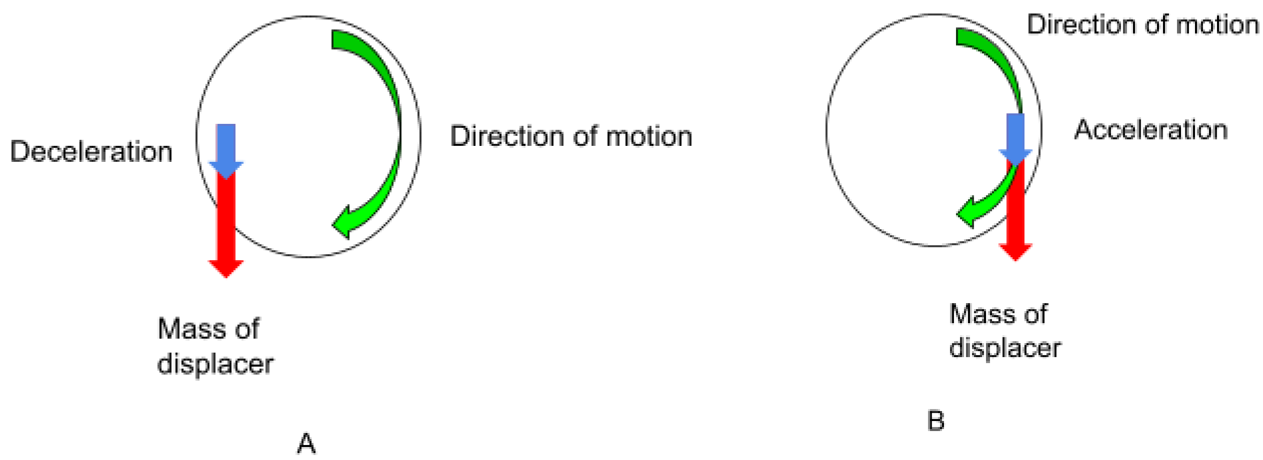

Figure 10: A: Deceleration position of displacer mass. B: Acceleration position of displacer mass.

In order to eliminate the acceleration and deceleration of the flywheel, a counter balance must be added to the displacer hub that offsets the moment of from the mass of the displacer. Ensuring that the moments on each side of the hub offset one another will make the flywheel spin at a constant speed throughout its rotation. Figure 11, shows the displacer and counterbalance moments.

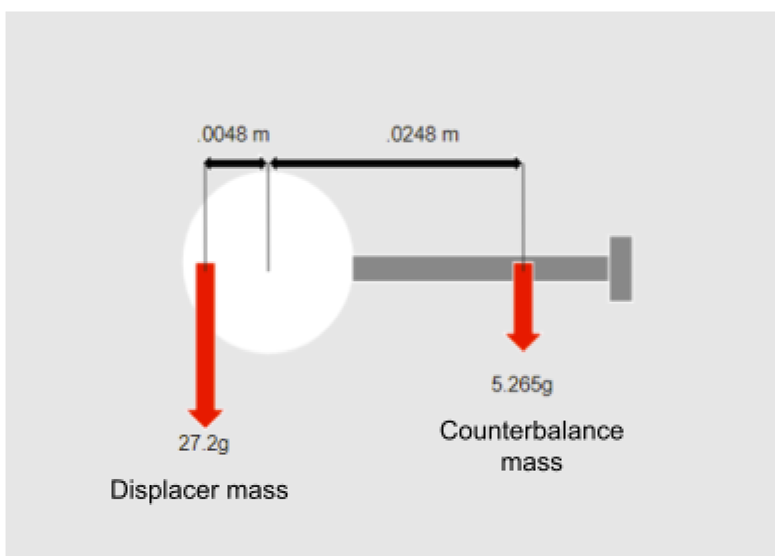

Figure 11: Mass and moment distance on each side of the displacer hub.

As shown in the figure above, the displacer mass is 27.2g positioned .0048m from the center of the hub. Following the torque Equation 2 [5],

$$\tau = F * d$$

Equation 2

where τ is torque, F is force, and d is distance, the torque from the displacer mass can be calculated as follows. $\tau = \frac{27.2\,[g]}{1000\,[g/kg]} * 9.81\,[m/s^2] *.0048\,[m] = .00128\,[N-m]$. To balance this, on the opposite side of the hub a mass of 5.265 g must be placed .0248m from the center. The torque for the counterbalance mass is calculated similarly.
$\tau = \frac{5.265\,[g]}{1000\,[g/kg]} * 9.81\,[m/s^2] *.0248\,[m] = .00128\,[N-m]$. Since the torques are equal on each side of the hub, the flywheel will no longer have acceleration or deceleration throughout its rotation and will spin at a constant velocity. To add the mass for the counterbalance, a 1.5” 8-32 bolt was threaded into a new hole opposite the displacer. Additionally, the 4-40 set screw was redrilled and tapped 90 degrees clockwise from its original position. Because the mass of the 8-32 screw was only 3.8g, an additional 1.465g was needed. A small Wisconsin logo “motion W” was 3D printed with this exact mass. Figure 12 shows the redesigned hub with counterbalance.

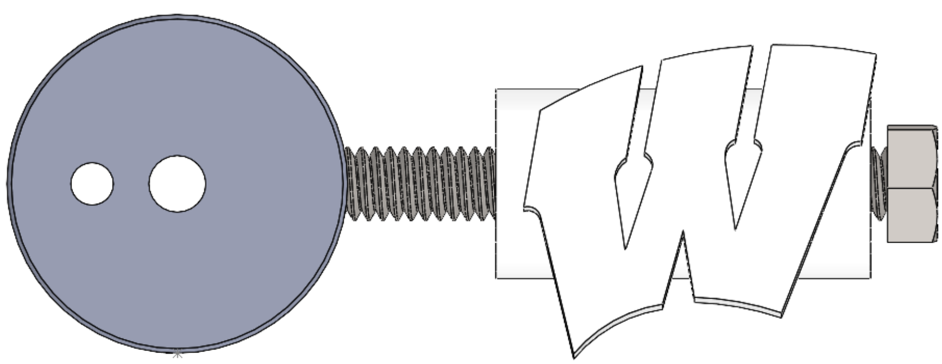

Figure 12: Redesigned displacer hub with counter balance.

Unfortunately, during assembly of the stirling engine, it was discovered that the motion W interfered with the links connecting the hub to the displacer. For this reason, the 3D printed motion W was removed but the 8-32 bolt remained. While the moments on each side of the hub are not completely equal, a similar torque on each side improved the even rotation of the flywheel.

Bolts Holding the Plates

One effective way to maximize the engine's performance is to maximize the temperature difference between the top and bottom plates. The larger the temperature difference, the higher the pressure difference on the displacer leading to faster engine speeds. One factor that decreases this temperature difference is conduction between the top and bottom plates of the engine through the bolts holding the two plates together. The heat transfer through the bolts is defined by Equation 3.

$$\dot{q}_{bolts} = N_{bolts} * k_{bolt} * A_c * \frac{1}{L} * (T_{RT} - T_{cold})$$

Equation 3

The baseline engine design had 12 stainless steel bolts with a thermal conductivity of 14 W/mK [8] resulting in total heat transfer of 0.851 W. From equation 1, the only two parameters that could be changed without permanently affecting the design of the engine or any of the components was the number of bolts and the thermal conductivity of the bolts, i.e. the bolts' material. The number of bolts was reduced by half, from 12 to 6, and the material was changed to Nylon Plastic to have a thermal conductivity of .25 W/mK [9]. The heat transfer through the 6 plastic bolts was calculated to be 0.0075 W, resulting in a 99.1% reduction in conductive heat transfer through the bolts. Overall, this change was very effective, especially given that it required no extra machining and that it was relatively inexpensive.

Testing Plan and Results

Describe how you tested the performance of your redesigned Stirling Engine. What were the results? What tests would you have liked to conduct, but could not because of time, technology, or knowledge constraints?

To test the performance of our redesigned Stirling Engine, we would measure the speed of the Stirling Engine using the Metronome app after each design change. If the speed showed a decrease in speed, we would revert the change to eliminate the changes that would slow down the engine. Table 4 displays how each design change affected the speed of the engine.

Table 4: Speed of engine as a result of design changes.

Design Change	**RPM**
Baseline Build	77
Cylinder Base	77 (no definitive change)

Flywheel	Not measured, expected ~ 70-80
Counter Balance	87
Linkages with shoulder bolts and sleeve bearings	155
Testing Day (sitting in sunlight)	212

Unfortunately, not all of our re-designed components were very successful with most of which having very little or no change in speed. However, the team managed to increase engine speed by roughly 175%, which is a very significant improvement.

One other test that we would have liked to conduct was a pressure test to identify fluctuations in pressure during the engine's cycle. By placing pressure transducers in between the two plates, one on the top plate and one on the bottom, the pressure could be measured and analyzed using data acquisition software. The method we are most familiar with is connecting two pressure transducers to a National Instruments myDAQ, recording the data using National Instruments LabView software, and EES for post-processing.

Summary

With a roughly 175% improvement in speed and almost perfect aesthetics/build quality scores, it was concluded that this redesign project was a success. In addition to making significant improvements in the competition aspect of the project, this project was also deemed successful due to the amount of experience and knowledge gained by taking part in it. The team members were able explore different manufacturing processes, experience the process of getting valuable customer information, and iterate over previous designs to optimize the engine. Table 5 displays the amount of time spent by each team member on the semester project.

Table 5: Time record for team members of Group J.

	Ahmed Khalil	Josh Richlen
Manufacturing baseline engine	20 hours	20 hours
Assembling baseline engine	3 hours	3 hours
Redesign research	20 hours	15 hours

Manufacturing new components	15 hours	18 hours
Testing redesign	4 hours	4 hours
Total time	**62 hours**	**60 hours**

Acknowledgments

We would like to thank the Team Lab Pro Staff, specifically: Jeff Rappe, Jay Bowe, Eric Ellefsen, and Mike Hughes. The Team Lab Pro Staff have been essential to the success of this project by walking us through many of the manufacturing processes required to manufacture the Stirling Engine. When redesigning the Stirling Engine, the Team Lab Pro Staff shared with us their wealth of experience which helped us turn our design ideas into real components that could be integrated into the Stirling Engine final design.

We would also like to thank Professor Pfefforkorn for offering the course and taking the time to ensure that the class ran smoothly even with the supply chain issues surrounding many of the components required for the assembly of the engine. In addition to managing the course, many of the material covered during lecture was vital to the success of the project. Everything from the engineering analysis of the engine to the design ideation was crucial to the development of a Stirling Engine that satisfied all of our customer's requirements.

Last but not least, we would like to thank Mr. Hemant, the Team Lab Student Staff and Makerspace Staff for assisting us with tasks such Laser Cutting the displacer, Waterjet Cutting the flywheel, 3D printing various components, and much more. This redesign was very involved with various components and it would not have been possible if it hadn't been for everyone mentioned above.

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Appendix

Appendix A - EES Calculations for heat transfer through bolts

File:Part10Redesign-JoshRichlen.EES 5/12/2022 5:51:45 PM Page 1
EES Ver. 10.833: #100: For use only by Students and Faculty, College of Engineering University of Wisconsin - Madison

```
l_s = .562*convert(in,m)                         "stroke length"
l_i = .625*convert(in,m)                         "length of small diameter"
D_o = .623*convert(in,m)                         "Outer Diameter of original part 10"
D_i = .31*convert(in,m)                          "inner diameter of original part 10"
rho_air = 1.204 [kg/m^3]                         "density of air at 20 C"
nu_air = 1.516e-5 [m^2/s]                        "kinematic viscosity of air at 20 C"
mu_air = 1.825e-5 [kg/m-s]                       "dynamic viscosity of air at 20 C"
g = 9.81 [m/s^2]                                 "g constant"
rpm = 77 [rev/min]                               "measured speed of Stirling Engine"
V_1 = rpm*l_s/.5[rev]*convert(m/min,m/s)         "velocity of air"
K_lam_contract = .38                             "loss coefficient of flow moving through a constriction Fig 6.22 White"
K_lam_expand = .58                               "loss coefficient of flow moving through an expansion Fig 6.22 White"

"Analysis"
Re_d_1 = rho_air*V_1*D_i/mu_air                  "reynolds number above cylinder base"
Re_d_2 = rho_air*V_2*D_i/mu_air                  "reynolds number through cylinder base"

"Both reynolds numbers are below 2300 indicating the flow is laminar throughout"

V_1*D_o^2 = V_2*D_i^2                            "flow rate is the same in a sealed pipe system"

f_1 = 64/Re_d_1                                  "friction factor in larger diameter pipe (power piston cylinder)"
f_2 = 64/Re_d_2                                  "friction factor in smaller diameter pipe (through constriction of part
10: cylinder base)"

DELTAh_A_B_contract = V_1^2/(2*g)*((f_1*l_s/D_o)+K_lam_contract) + V_2^2/(2*g)*(f_2*l_i/D_i)          "head loss of
contraction (downward stroke of piston)"
DELTAh_A_B_contract = DELTAP_contract/rho_air/g                                                         "pressure drop
through contraction"

DELTAh_A_B_expand = V_1^2/(2*g)*((f_1*l_s/D_o)+K_lam_expand) + V_2^2/(2*g)*(f_2*l_i/D_i)              "head loss of
expansion (upward stroke of piston)"
DELTAh_A_B_expand = DELTAP_expand/rho_air/g                                                             "pressure drop
through expansion"

DELTAh_A_B = V_1^2/(2*g)*(f_1*(l_s+l_i)/D_o)                                                            "head loss
through redesigned cylinder base"
DELTAh_A_B = DELTAP_tube_down/rho_air/g                                                                 "pressure drop
on downward stroke"
DELTAh_A_B = DELTAP_tube_up/rho_air/g                                                                   "pressure drop
on upward stroke"
```

$l_s = 0.562 \cdot \left| 0.0254 \cdot \frac{m}{in} \right|$ *stroke length*

$l_i = 0.625 \cdot \left| 0.0254 \cdot \frac{m}{in} \right|$ *length of small diameter*

$D_o = 0.623 \cdot \left| 0.0254 \cdot \frac{m}{in} \right|$ *Outer Diameter of original part 10*

$D_i = 0.31 \cdot \left| 0.0254 \cdot \frac{m}{in} \right|$ *inner diameter of original part 10*

$\rho_{air} = 1.204$ [kg/m^3] *density of air at 20 C*

$\nu_{air} = 0.00001516$ [m^2/s] *kinematic viscosity of air at 20 C*

$\mu_{air} = 0.00001825$ [kg/m-s]

dynamic viscosity of air at 20 C

$g = 9.81$ [m/s²] *g constant*

$rpm = 77$ [rev/min] *measured speed of Stirling Engine*

$V_1 = rpm \cdot \dfrac{l_s}{0.5\ [rev]} \cdot \left| 0.016666667 \cdot \dfrac{m/s}{m/min} \right|$ *velocity of air*

$K_{lam,contract} = 0.38$ *loss coefficient of flow moving through a constriction Fig 6.22 White*

$K_{lam,expand} = 0.58$ *loss coefficient of flow moving through an expansion Fig 6.22 White*

Analysis

$Re_{d,1} = \rho_{air} \cdot V_1 \cdot \dfrac{D_i}{\mu_{air}}$ *reynolds number above cylinder base*

$Re_{d,2} = \rho_{air} \cdot V_2 \cdot \dfrac{D_i}{\mu_{air}}$ *reynolds number through cylinder base*

Both reynolds numbers are below 2300 indicating the flow is laminar throughout

$V_1 \cdot D_o^2 = V_2 \cdot D_i^2$ *flow rate is the same in a sealed pipe system*

$f_1 = \dfrac{64}{Re_{d,1}}$ *friction factor in larger diameter pipe (power piston cylinder)*

$f_2 = \dfrac{64}{Re_{d,2}}$ *friction factor in smaller diameter pipe (through constriction of part 10: cylinder base)*

$\Delta h_{A,B,contract} = \dfrac{V_1^2}{2 \cdot g} \cdot \left[f_1 \cdot \dfrac{l_s}{D_o} + K_{lam,contract} \right] + \dfrac{V_2^2}{2 \cdot g} \cdot f_2 \cdot \dfrac{l_i}{D_i}$ *head loss of contraction (downward stroke of piston)*

$\Delta h_{A,B,contract} = \dfrac{\Delta P_{contract}}{\rho_{air} \cdot g}$ *pressure drop through contraction*

$\Delta h_{A,B,expand} = \dfrac{V_1^2}{2 \cdot g} \cdot \left[f_1 \cdot \dfrac{l_s}{D_o} + K_{lam,expand} \right] + \dfrac{V_2^2}{2 \cdot g} \cdot f_2 \cdot \dfrac{l_i}{D_i}$ *head loss of expansion (upward stroke of piston)*

$\Delta h_{A,B,expand} = \dfrac{\Delta P_{expand}}{\rho_{air} \cdot g}$ *pressure drop through expansion*

$\Delta h_{A,B} = \dfrac{V_1^2}{2 \cdot g} \cdot f_1 \cdot \left[\dfrac{l_s + l_i}{D_o} \right]$ *head loss through redesigned cylinder base*

$\Delta h_{A,B} = \dfrac{\Delta P_{tube,down}}{\rho_{air} \cdot g}$ *pressure drop on downward stroke*

$\Delta h_{A,B} = \dfrac{\Delta P_{tube,up}}{\rho_{air} \cdot g}$ *pressure drop on upward stroke*

File:Part10Redesign-JoshRichlen.EES 5/12/2022 5:51:45 PM Page 3
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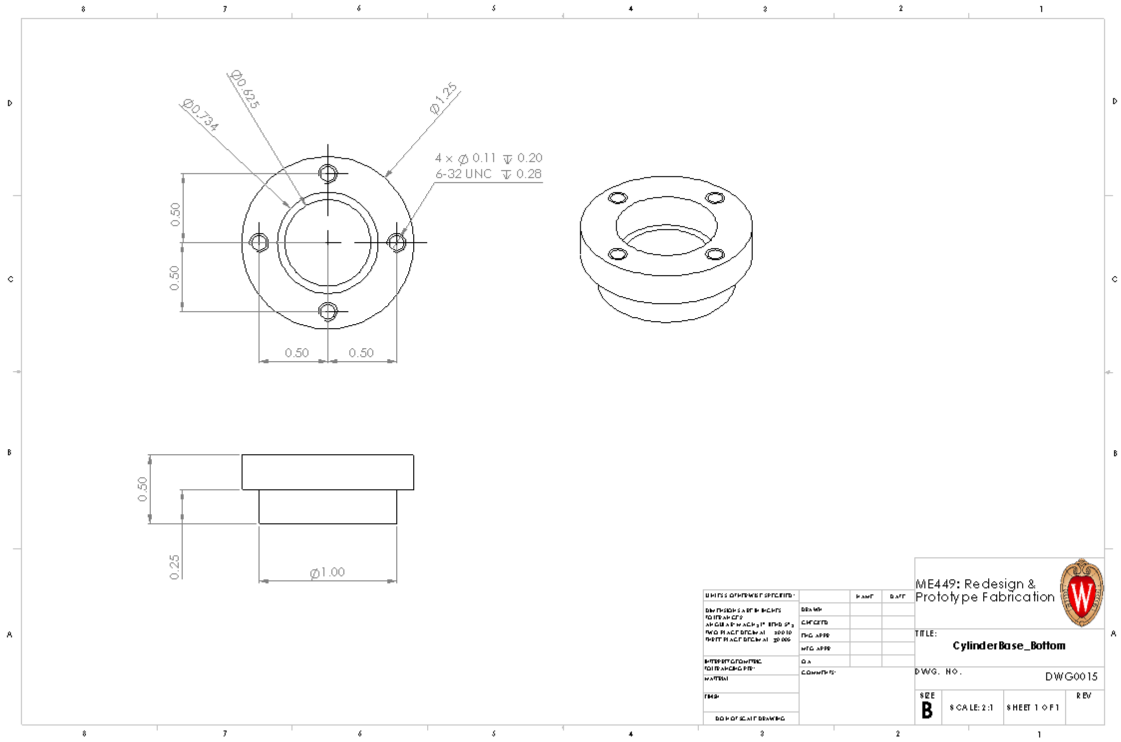
SOLUTION

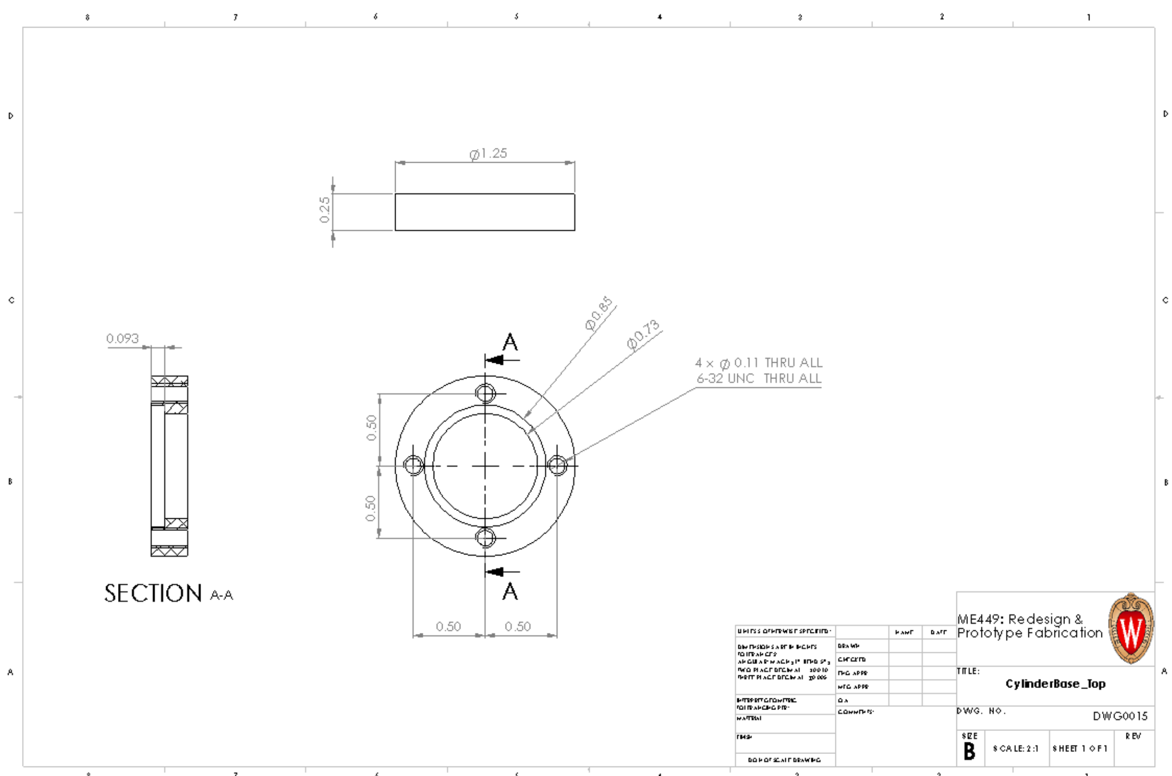
Unit Settings: SI C kPa kJ mass deg

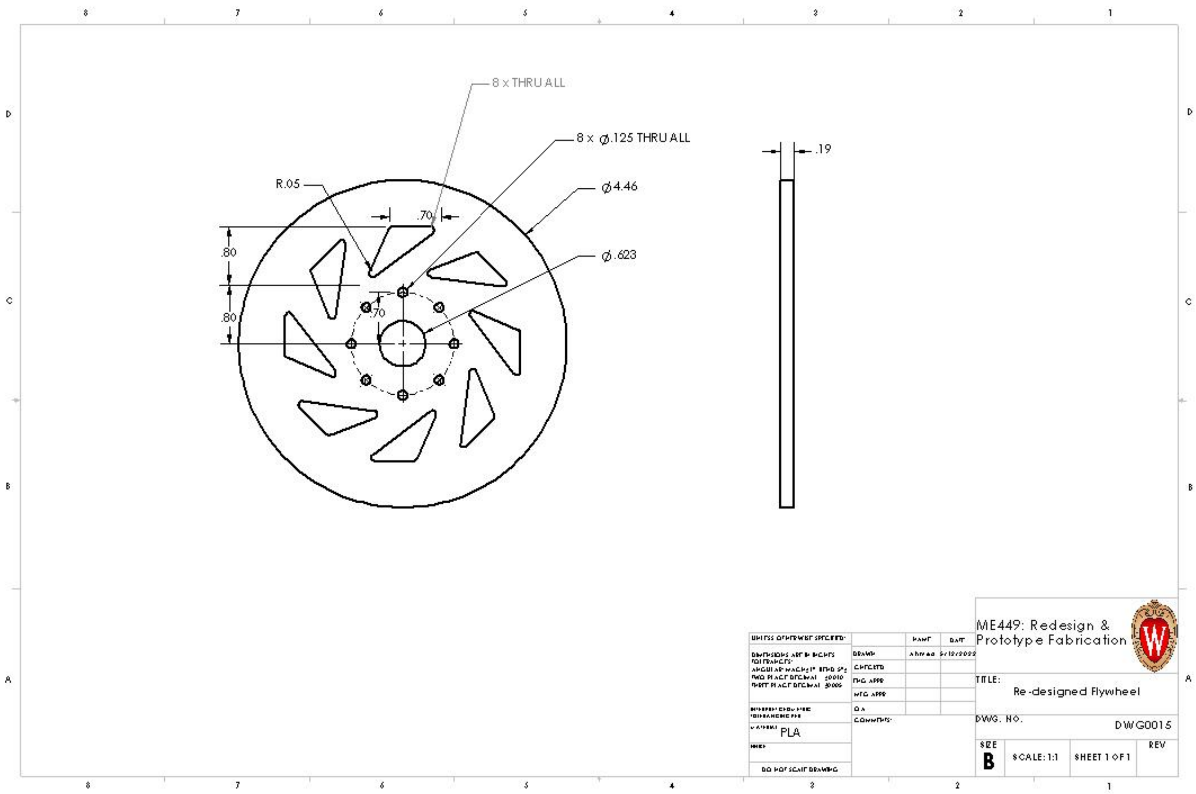
$\Delta h_{A,B}$ = 0.0004384 [m]	$\Delta h_{A,B,contract}$ = 0.002107 [m]
$\Delta h_{A,B,expand}$ = 0.002121 [m]	$\Delta P_{contract}$ = 0.02489 [Pa]
ΔP_{expand} = 0.02505 [Pa]	$\Delta P_{tube,down}$ = 0.005177 [Pa]
$\Delta P_{tube,up}$ = 0.005177 [Pa]	D_i = 0.007874 [m]
D_o = 0.01582 [m]	f_1 = 3.363
f_2 = 0.8326	g = 9.81 [m/s^2]
$K_{lam,contract}$ = 0.38	$K_{lam,expand}$ = 0.58
l_i = 0.01588 [m]	l_s = 0.01427 [m]
μ_{air} = 0.00001825 [kg/m-s]	ν_{air} = 0.00001516 [m^2/s]
$Re_{d,1}$ = 19.03	$Re_{d,2}$ = 76.87
ρ_{air} = 1.204 [kg/m^3]	rpm = 77 [rev/min]
V_1 = 0.03664 [m/s]	V_2 = 0.148 [m/s]

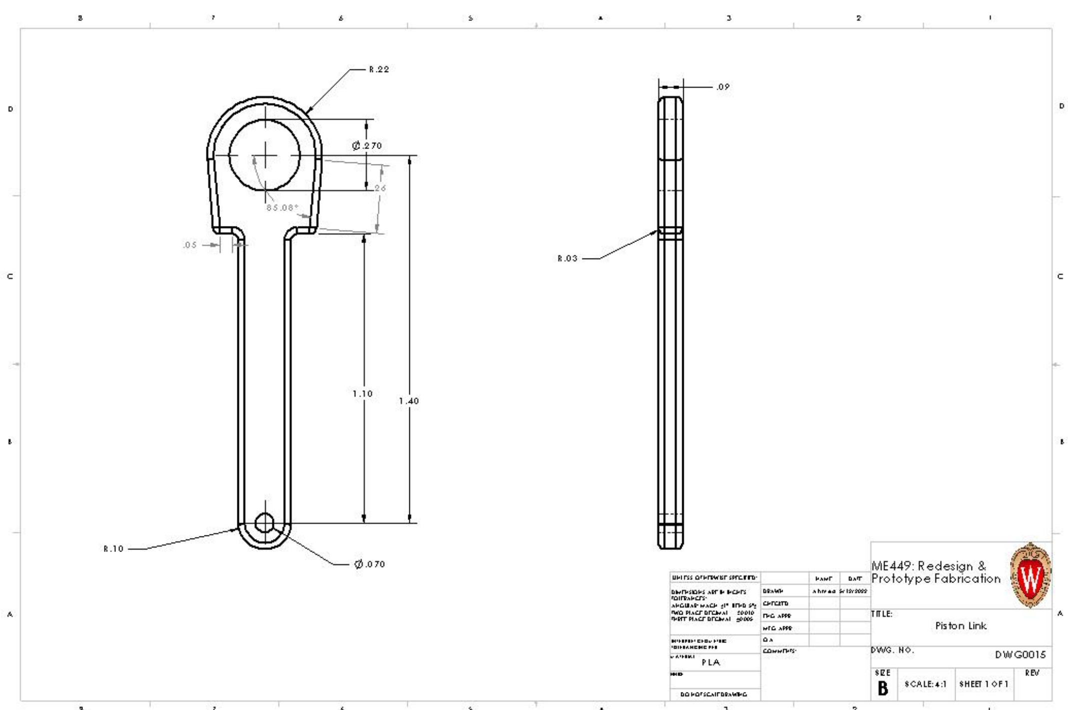
No unit problems were detected.

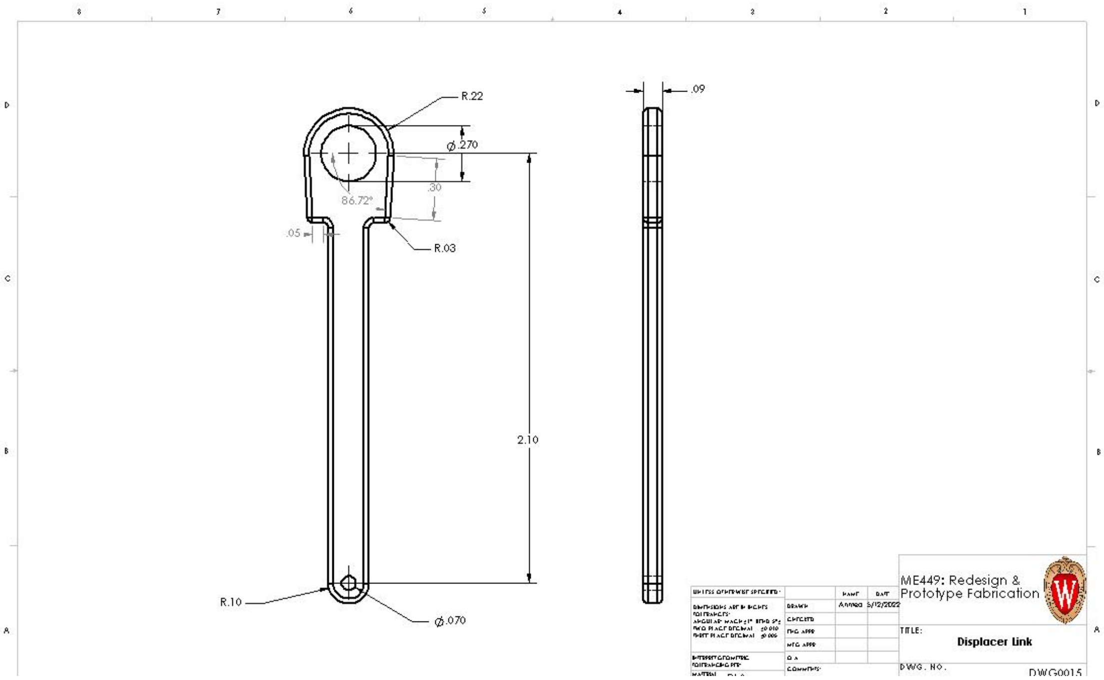
Appendix B - Part Drawings






8 x THRU ALL
8 x ⌀.125 THRU ALL
.19
R.05
⌀4.46
.70
.80
.80
⌀.623
.70
ME449: Redesign & Prototype Fabrication
TITLE:
Re-designed Flywheel
DWG. NO.
DWG0015
SIZE
B
SCALE: 1:1
SHEET 1 OF 1
REV
PLA
DO NOT SCALE DRAWING


R.22
.09
⌀.270
.26
85.08°
.05
R.03
1.10
1.40
R.10
⌀.070
ME449: Redesign & Prototype Fabrication
TITLE:
Piston Link
DWG. NO.
DWG0015
SIZE
B
SCALE: 4:1
SHEET 1 OF 1
REV
PLA
DO NOT SCALE DRAWING


R.22
Ø.270
.30
86.72°
.05
R.03
2.10
R.10
Ø.070
.09
ME449: Redesign &
Prototype Fabrication
TITLE:
Displacer Link
DWG. NO.
DWG0015